Test Review

Do it strong.
A biology professor responds to some student questions by e-mail. The probability model below describes the number of e-mails that the professor may receive from students during a day.

<table>
<thead>
<tr>
<th>E-mails received</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
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<td>Probability</td>
<td>0.05</td>
<td>0.10</td>
<td>0.20</td>
<td>0.25</td>
<td>0.30</td>
<td>0.10</td>
</tr>
</tbody>
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a. How many e-mails should the professor expect to receive each day?

b. What is the standard deviation?
A biology professor responds to some student questions by e-mail. The probability model below describes the number of e-mails that the professor may receive from students during a day.

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c. If it takes the professor an average of ten minutes to respond to each e-mail, how much time should the professor expect to spend responding to student e-mails each day?
Let $X$ = number of e-mails received.

$E(X) = 0(0.05) + 1(0.10) + 2(0.20) + 3(0.25) + 4(0.30) + 5(0.10) = 2.95$ e-mails per day

$Var(X) = (0 - 2.95)^2 (0.05) + (1 - 2.95)^2 (0.10) + (2 - 2.95)^2 (0.20) + (3 - 2.95)^2 (0.25) + (4 - 2.95)^2 (0.30) + (5 - 2.95)^2(0.10) = 1.7475$

$SD(X) = \sqrt{1.7475} = 1.32$ e-mails per day

Mean number of minutes: $(2.95)(10)=29.5$ min.
The American Veterinary Association claims that the annual cost of medical care for dogs averages $100 with a standard deviation of $30, and for cats averages $120 with a standard deviation of $35.

a. Find the expected value for the annual cost of medical care for a person who has one dog and one cat.

b. Find the standard deviation for the annual cost of medical care for a person who has one dog and one cat.
The American Veterinary Association claims that the annual cost of medical care for dogs averages $100 with a standard deviation of $30, and for cats averages $120 with a standard deviation of $35.

c. Suppose that a couple owns four dogs.
   i. Find the expected value for the annual cost of medical care for the couple's dogs.
   ii. Find the standard deviation for the annual cost of medical care for the couple's dogs.
• a. \( E(D + C) = E(D) + E(C) = \$100 + \$120 = \$220 \)

• b. \( Var(D + C) = Var(D) + Var(C) = 30^2 + 35^2 = 2125, \) so \( SD(D + C) = \sqrt{2125} = \$46.10 \)

• c. i. \( E(D_1 + D_2 + D_3 + D_4) = \$100 + \$100 + \$100 + \$100 = \$400 \)

• ii. \( Var(D_1 + D_2 + D_3 + D_4) = 30^2 + 30^2 + 30^2 + 30^2 = 3600, \) so \( SD(D_1 + D_2 + D_3 + D_4) = \sqrt{3600} = \$60 \)
A small business just leased a new computer and color laser printer for three years. The service contract for the computer offers unlimited repairs for a fee of $100 a year plus a $25 service charge for each repair needed. The company's research suggested that during a given year 86% of these computers needed no repairs, 9% needed to be repaired once, 4% twice, 1% three times, and none required more than three repairs.
• Find the expected number of repairs this kind of computer is expected to need each year.
• Find the standard deviation of the number of repairs each year.
• What are the mean and standard deviation of the company’s annual expense for the service contract?
How many times should the company expect to have to get this computer repaired over the three-year term of the lease?
\begin{itemize}
  \item \( E(X) = 0(0.86) + 1(0.09) + 2(0.04) + 3(0.01) = 0.20 \) repairs
  \item \( Var(X) = (0 - 0.2)^2(0.86) + (1 - 0.2)^2(0.09) + (2 - 0.2)^2(0.04) + (3 - 0.2)^2(0.01) = 0.30 \)
  \hspace{1cm} \( SD(X) = 0.55 \) repairs
  \item Let \( C = 100 + 25X; \ E(C) = 100 + 25(0.20) = $105; \)
  \hspace{1cm} \( SD(C) = 25(0.55) = $13.69 \)
  \item \( E(X_1 + X_2 + X_3) = 0.20 + 0.20 + 0.20 = 0.60 \) repairs.
\end{itemize}
The American Red Cross says that about 11% of the U.S. population has Type B blood. A blood drive is being held at your school.

How many blood donors should the American Red Cross expect to collect from until it gets a donor with Type B blood?

What is the probability that the tenth blood donor is the first donor with Type B blood?
What is the probability that exactly 2 of the first 20 blood donors have Type B blood?

What is the probability that at least 2 of the first 10 blood donors have Type B blood?
The blood drive has a total of 150 donors. Assuming this is a typical number of donors for a school blood drive, what would be the mean and standard deviation of the number of donors who have Type B blood?
This is a Geometric model with $p = 0.11$.

Expected value: $\mu = \frac{1}{p} = \frac{1}{0.11} = 9.1$ donors

- $P(9 \text{ not Type B, Type B on 10th}) = (0.89)^9(0.11) = 0.0385$

$P(\text{exactly 2 out of 20}) = P(X = 2) = \binom{20}{2}(0.11)^2(0.89)^{18} = 0.2822$

$P(X \geq 2) = 1 - P(X \leq 1) = 1 - \text{BinomCDF}(10, 0.11, 1)$
Using the Binomial model,
Mean: $\mu = np = (150)(0.11) = 16.5$
Standard deviation: $\sigma = \sqrt{npq} = \sqrt{(150)(0.11)(0.89)} = 3.83$
• A young boy is fishing off the end of a dock. He estimates that for one out of every 15 times he casts his line, he gets at least a nibble from a curious fish. He is going to cast his line 50 times before he switches to toad hunting.

• Verify that this scenario satisfies all four conditions for a binomial scenario.
What is the probability that the fisherman will get 5 nibbles on his line.

What are the mean and standard deviation of the number of successes he will have out of the 50 attempts?

What is the probability that he will cast his line without success 20 times before finally succeeding the 21st time?
Suppose that our fearless fisherman goes out early one morning and casts a total of 250 times. What are the mean and standard deviation of the number of times he should receive a nibble on his line?

If he gets only 5 nibbles on his line, is that a signal that he is having less success than usual? Justify your answer.
• We are counting the number of successes or failures—nibble on the line or not. We are assuming that $\frac{1}{15}$ is the fixed probability of getting a nibble and that each cast is independent. We are testing out of 50 trials, our set number of trials.
Test Review Solutions

\[ P(x = 5) = \binom{50}{5} \left( \frac{1}{15} \right)^5 \left( \frac{14}{15} \right)^{45} = 0.125 \]

\[ np = 50 \left( \frac{1}{15} \right) = 3.33 \text{ nibbles; } sd = \sqrt{50 \left( \frac{1}{15} \right) \left( \frac{14}{15} \right)} = 1.76 \text{ nibbles} \]

\[ \left( \frac{14}{15} \right)^{20} \left( \frac{1}{15} \right) = 0.01677 \]

\[ np = 250 \left( \frac{1}{15} \right) = 16.67 \text{ nibbles} \]

\[ sd = \sqrt{250 \left( \frac{1}{15} \right) \left( \frac{14}{15} \right)} = 3.94 \]

\[ \frac{5 - 16.67}{3.94} = -2.96 \]

Three standard deviations away means he should try a new bait.